Abstract for a *Lichtenberg-Kolleg Kurzzeit*-Fellowship

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Hilbert as outreacher: the genesis of *Anschauliche Geometrie*

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1 Summary

These four pages outline a research project for a stay of one month (proposed for the period **1st–30th September 2013**) at the *Lichtenberg-Kolleg* in Göttingen. My main goal is to make a survey of original materials held in the collections of the University of Göttingen related to the genesis of *Anschauliche Geometrie*, a classic book [2] by David Hilbert and Stephan Cohn-Vossen aimed at making geometry accessible to the layperson. The central issue I want to clarify is the extent to which the original project that evolved into this celebrated book, which was a lecture course delivered by Hilbert in 1920/21 at Göttingen, was already aimed at a very general (and possibly non-mathematical) audience, and can be regarded as an early endeavour in outreach or popularisation of science.

2 Some background

It is very well known that the University of Göttingen played a pivotal role in the development of mathematics in the early twentieth century, attracting international researchers and ambitious students eager to interact with some of the most brilliant mathematical minds of the time, who happened to have Göttingen as their home. There was a climate of intense intellectual activity that went rather remarkably unscathed through the First World War, and which reached its peak right before the rise of the Nazi party and its seizure of power in 1933. Under the *Gesetz zur Wiederherstellung des Berufbeamtentums* enforced in that year, a great number of the most prominent academics at the Mathematical Institute in Göttingen were dispersed within the space of just a few months — either by force of their Jewish connections, or out of ideological dissent with the new regime.

In these golden years of Göttingen (and German) mathematics, a spirit of interdisciplinarity that was quite unique at the time pervaded the life of the Mathematical Institute. This was in part due to the presence of key figures such as Felix Klein and David Hilbert, and later Richard Courant, Hermann Weyl and Emmy Noether, who not only were generalists with a broad spectrum of scientific interests (transcending mathematics itself), but who also fostered a sense of community with activities that went beyond lecture and

office times. Many were the occasions for scientific discussions in a relaxed atmosphere, from regular dinners to walks in the countryside. The impact of this community on the shaping of mathematics as we know it today was tremendous, and it also served as a nest for young mathematicians who went on to make successful careers elsewhere.

There are scattered accounts on how the teaching of mathematics was conducted at Göttingen in those days. The approach to lecturing was very diverse among the professors and sometimes quite original [4]. Hilbert distinguished himself as a very charismatic pedagogue; he imparted a certain degree of experimentation to his lectures, and sometimes embarked in somewhat unconventional endeavours. One example was his decision to offer a one-semester lecture course (held in the Winter Semester of 1920/21) of four hours per week entitled *Anschauliche Geometrie*: that is, intuitive geometry, made directly accessible with minimal technical prerequisites by one of its most renowned practicians. One may presume that this course might have had a direct practical motivation: this was the time when many students returned from the war after a long absence, and they would be more receptive to a course that was in a sense less demanding; or it might have been intended as a kind of light antidote to the intuitionist programme initiated by Brouwer that was gathering sympathisers in Göttingen, and to which Hilbert was very much opposed. In any case, this was by no means an ephemeral venture: the course was replicated twice by Hilbert himself, and then continued by one of his younger students, Walther Rosemann. Hilbert's project eventually evolved into a book [2] which was put together by a young geometer also resident in Göttingen, Stephan Cohn-Vossen, who died prematurely shortly after the publication. This famous book has been widely acknowledged as one of the highest achievements in mathematical exposition to the general public, and it provides the layperson with an accurate (and timeless) impression of what the spirit of mathematics is about. In Hilbert's own words, in his preface of 1932 [2, p. XVIII]: *Das Buch soll dazu dienen, die Freude an der Mathematik zu mehren, indem es dem Leser erleichtert, in das Wesen der Mathematik einzudringen, ohne sich einem beschwerlichen Studium zu unterziehen.* Further on, he adds this often cited line [ibid.]: *Der Leser soll [...] in dem großen Garten der Geometrie spazieren geführt werden, und jeder soll sich einen Strauß pflücken können, wie er ihm gefällt.*

The first edition of this book, published in 1932 by Springer-Verlag, enjoyed immediate acclaim by the mathematical community. The preface to the second German edition of 1991, by the French geometer Marcel Berger, starts with the sentence [2, p. VII] : *Le livre qui suit est tout à fait extraordinaire.* Berger goes on to digress on why such a book has been enjoying such longevity, and points out how traits of Hibert's own personality stand out from what one can refer to as the narrative; at stretches, the composition almost confers to him, in the eyes of the reader, the aura of a mathematical conjurer. It is quite unfortunate that, for the majority of the public, the ingenuity and remarkable balance of his masterful exposition is only accessible via a very average English translation [3] of 1952, which runs short of the beautiful prose of Cohn-Vossen and contains some mathematical misunderstandings. Even the title given to this translation, *Geometry and the Imagination*, diverts the readers from the main aspiration of the book, which was so transparently conveyed by its original title: as a matter of fact, this work appeals very essentially to intuition, and it guides the reader through some remarkably intricate geometrical constructions with the help of copious diagrams and photographs of mathematical models (361 in total). Most of these models (in

plaster, cardboard or metal) are still kept at the exceptionally rich collection of mathematical models and instruments of the Mathematical Institute at Göttingen.

3 General motivation

As we approach the centenary of the first concept of *Anschauliche Geometrie* (in the form of the course delivered at Göttingen by its senior author, which formed the seed for the book), there is an opportunity to look back once again and try to understand at depth the circumstances that prompted Hilbert to embark in such a project. It is also appropriate to examine this masterful work from a modern scientific and pedagogic perspective, since many of the topics touched upon in the book, pertaining to geometry but also to other branches of mathematics, have had many developments over almost a century, or have been discarded from the foundational mathematical syllabus.

The main purpose of my stay is to take advantage of the facilities at the University of Göttingen (in particular, access to the university archives, to the collection of mathematical models, and to original local newspapers from Hilbert's time) to obtain new insights on the genesis of this unique piece of work, and about the attention that it received from the wider community in the city of Göttingen. I regard this as necessary field work which is preliminary to a more ambitious, long-term outreach project that I have in mind: producing a comprehensive critical edition of *Anschauliche Geometrie* that can be readily used by a modern audience, in particular as a work tool by beginning students of mathematics and the sciences.

I have started preparing a translation of the book, directly from the German text into Portuguese, that might evolve into a seed or pilot version of such a new edition. It is expected to contain, apart from the translation of the original text: expository mathematical comments complementing and updating individual sections, a refreshment of the original photographs into high-definition versions of the same models in colour, extensive biographical notes, as well as a historical account of the scientific climate in Göttingen at the zenith of its mathematical prominence. I have already initiated contact with the board of editorial projects of the *Sociedade Portuguesa de Matemática* in Lisbon, which holds a special protocol for copyright and translations into Portuguese with Springer-Verlag, and their interest in this initiative was manifest. As my extended project comes to a term, this first Portuguese edition may well serve as a prototype for critical editions of the book in other languages (e.g. in German, with improved critical notes accompanying the original text, and/or in English) provided that sufficient interest arises from publishers and copyright owners. The time-frame I am setting for this project makes it likely to be ready well before the 100th anniversary of Hilbert's course at Göttingen in 2020.

4 Activities planned

Most of the work will be conducted in the archives and libraries of the University of Göttingen. In particular, I want to examine the *Handschriften, Autographen, Nachlässe, Sonderbestände* collection of the *Niedersächsische Staats- und Universitätsbibliothek Göttingen*, which holds

an early version of the notes of Hilbert's original course (Cod.Ms.D.Hilbert 563, written by Rosemann), as well as notes for earlier courses held by Hilbert at the Mathematical Institute which inspired sections of *Anschauliche Geometrie*. The catalogue [1] of manuscripts related to Hilbert lists other writings by Rosemann (Cod.Ms.D.Hilbert 713+714), some of which still unclassified, that may be of interest to my research project. I would also like to have access to the *Sammlung Mathematischer Modelle und Instrumente* of the Mathematical Institute; provided that permission is granted, I am planning to take new colour photographs in digital format of all the models appearing in the book that are still part of the collection, with a view towards refreshing (in the Portuguese edition mentioned above) the evocative but outdated original photographs reproduced in editions of the book currently available. Finally, I will use part of my time at Göttingen to make contact with Springer-Verlag regarding copyright issues and other practical matters related to the on-going Portuguese translation and edition.

References

[1] M. Fimpel (Bearb.): *Spezialinventar zur Geschichte der Mathematik und Naturwissenschaften an der Universitt Göttingen von 1880–1933 : ein Führer zu den archivalischen Quellen*. Göttingen, 2002

[2] D. Hilbert und S. Cohn-Vossen: *Anschauliche Geometrie*. Zweite Auflage, Springer-Verlag, 1991

[3] D. Hilbert and S. Cohn-Vossen: *Geometry and the Imagination* (translated by P. Nemenyi). Second Edition, Chelsea Publishing Company, 1983

[4] C. Reid: *Hilbert — Courant*. Springer-Verlag, 1986